

Date: 10/15/18

Chp: Chp. 2:4 → Avg. Rates of
Change & Tangent Lines

Obj: Find avg rates of change, tangent
to a curve, slope of a curve, & normal
to a curve.

1) Drive 135 mi in 2.5 hrs,
what was your avg. speed?

54 mph

2) If cost \$260 to buy 13 books,
what was avg cost per book?

\$20

Ex. 1

If $f(2) = 3$ & $f(8) = -15$,
what's the avg. rate of change?

$(2, 3), (8, -15)$

$$\frac{\Delta y}{\Delta x} = \boxed{-3}$$

Ex. 2

$$f(x) = 0.25x^2$$

what is the avg. rate of
change over....

$$a) \left[\begin{array}{c} x \\ -2 \\ 1 \end{array}, \begin{array}{c} x \\ 4 \\ 4 \end{array} \right]$$

$$\frac{\Delta y}{\Delta x} = \frac{4-1}{4+2} = \frac{3}{6} = \frac{1}{2}$$

$$0.25(-2)^2 \quad 0.25(4)^2$$

$$b) \left[\begin{array}{c} -4 \\ 4 \end{array}, \begin{array}{c} 2 \\ 1 \end{array} \right]$$

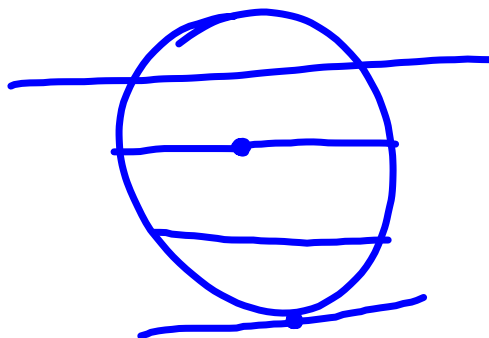
$$0.25(-4)^2 \quad 0.25(2)^2$$

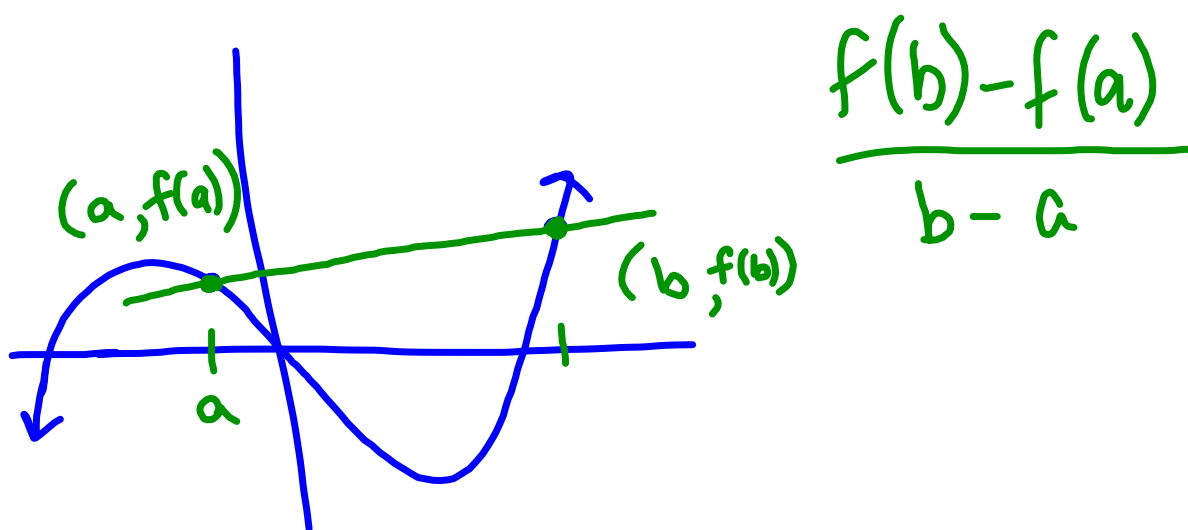
$$\frac{4-1}{-4-2} = \frac{3}{-6} = -\frac{1}{2}$$

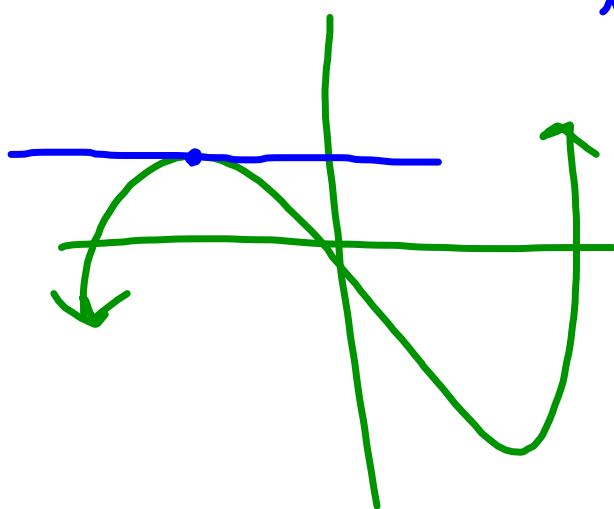
$$c) \left[\begin{array}{c} -1 \\ \frac{1}{4} \end{array}, \begin{array}{c} 1 \\ \frac{1}{4} \end{array} \right]$$

$$\frac{\frac{1}{4} - \frac{1}{4}}{\quad} = \boxed{0}$$

$$d) [-8, 0]$$







* The instantaneous rate of change is the slope of the tangent line to the curve.

Slope of a Curve @ a pt (aka slope of a tangent line)

The slope of a tangent to (or the instantaneous rate of change of) the curve $y = f(x)$ @ the pt. $P(a, f(a))$ is the number

$$m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists.

1) $f(a+h)$

2) $f(a)$

3) plug in # h is approaching & simplify.

Ex. 3 - Find the avg. rate of change over the given interval & the instantaneous rate of change @ Pt. P.

a) $f(x) = 2x^2 + 3x$ $[-1, 3]$ $x=2$

$$\text{avg} = \frac{27 + (3) - 28}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

instant = $f'(2+h)$

$$\begin{aligned} &2(2+h)^2 + 3(2+h) \\ &2(4 + 4h + h^2) + 3(2+h) \\ &8 + 8h + 2h^2 + 6 + 3h \\ &14 + 11h + 2h^2 \end{aligned}$$

② $f(2)$

$$\begin{aligned} &2(2)^2 + 3(2) \\ &14 \end{aligned}$$

$$\frac{14 + 11h + 2h^2 - 14}{h}$$

$$\frac{2h^2 + 11h}{h} = \cancel{h}(2h + 11) = 2h + 11$$

$$\lim_{h \rightarrow 0} 2h + 11 = 11$$

b) $g(x) = 4 - x^2$ $[-2, 3]$ $x=1$

$$\text{avg} = \frac{-5 - 0}{3 - (-2)} = \frac{-5}{5} = -1$$

instant = $f'(1+h)$

$$\begin{aligned} &4 - (1+h)^2 \\ &4 - (1 + 2h + h^2) \\ &4 - 1 - 2h - h^2 \\ &3 - 2h - h^2 \end{aligned}$$

$f(1)$

$$\begin{aligned} &4 - (1)^2 \\ &3 \end{aligned}$$

$$\frac{3 - 2h - h^2 - 3}{h} = \frac{-2h - h^2}{h} = \cancel{h}(-2 - h)$$

$$\lim_{h \rightarrow 0} -2 - h = -2$$

c) $f(x) = \frac{3}{x+2}$ $[-1, 3]$ $x=1$

$(-1, 3), (3, \frac{3}{5})$

$$\frac{\frac{3}{5} - 3}{3 - (-1)} = \frac{-\frac{12}{5}}{4} = \frac{-12}{20} = \frac{-3}{5}$$

$f(1+h)$

$$\frac{3}{(1+h)+2} = \frac{3}{3+h}$$

$f(1)$

$$\frac{3}{1+2} = \frac{3}{3} = 1$$

$$\lim_{h \rightarrow 0} \frac{\frac{3}{3+h} - 1}{h} = \frac{\frac{3 - (3+h)}{3+h}}{h} = \frac{\frac{-h}{3+h}}{h} = \frac{-1}{3+h}$$

$$\lim_{h \rightarrow 0} \frac{-1}{3+h} = \frac{-1}{3}$$

d) $f(x) = x^2 - 2x$ $[-5, 4]$ $x=2$

$m=2$

$y = mx + b$ $y = 2x - 4$

$0 = 2(2) + b$

$-4 = b$

$\perp m = -\frac{1}{2}$ $y = -\frac{1}{2}x + 1$

$0 = -\frac{1}{2}(2) + b$

$1 = b$

Ex. 4 - Avg. rate of change.

a) $g(x) = e^x$ $[0, 2]$
1 7.39

$$\frac{7.39 - 1}{2 - 0} = \frac{6.39}{2} = \textcircled{3.195}$$

b) $h(x) = \sqrt{x-2}$ $[6, 27]$
2 5

$$\frac{5-2}{27-6} = \frac{3}{21} = \textcircled{\frac{1}{7}}$$

Homework:

p. 92 (#1-19 odds, 22-32)